Case I. *Conservative System :*

We consider a conservative system and use above set of equations (8) to obtain Lagrange's equations of motion. For a conservative system, forces $\mathbf{F}_i$ are derivable from potential function V, which is the function of coordinates only. That is $V = V(\mathbf{r})$. Then

$$\mathbf{F}_i = -\nabla_i V = -\frac{\partial V}{\partial \mathbf{r}_i}$$

Then generalised force can be expressed as

$$Q_j = \sum_i \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} = -\sum_i \nabla_i V \cdot \frac{\partial \mathbf{r}_i}{\partial q_j}$$

$$= -\sum_i \frac{\partial V}{\partial \mathbf{r}_i} \cdot \frac{\partial \mathbf{r}_i}{\partial q_j}$$

$$= -\frac{\partial V}{\partial q_j}.$$

Eqs. (8) are now

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] = -\frac{\partial V}{\partial q_j}$$

or

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial (T-V)}{\partial q_j}\right] = 0$$

or

$$\left[\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial (T-V)}{\partial q_j}\right] = 0,$$

since V is not a function of $\dot{q}_j$. Recognising $(T - V)$ as L, the Lagrangian for the conservative system, equations become

$$\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j}\right] = 0, \qquad j = 1, 2, \ldots\ldots n \qquad \ldots (9)$$

which are known as Lagrange's equations of motion (of the second kind) for conservative system.

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1. Name : Dr. Vinay K Singh
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