Case II. ***Non-conservative System :***

If potentials are velocity dependent, called generalised potentials, then though the system is not conservative, yet the above form of Lagrange's equations can be retained provided Q_j, the components of generalised force, are obtained (see eq. 3 of art. 2.6) from a function $U(q_j, \dot{q}_j)$, called generalised potential, such that

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right). \qquad \dots (10)$$

Putting this value of Q_j in equation (8), we get

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right)$$

or

$$\frac{d}{dt}\left(\frac{\partial (T-U)}{\partial \dot{q}_j}\right)-\frac{\partial}{\partial q_j}(T-U)=0.$$

If we put Lagrangian $L = T - U$ then above equation becomes

$$\sum_j\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}\right]=0,$$

which is exactly of the same form as eq. (9). An example of such a type will follow in the calcula[tion] of Lagrangian for the case of electromagnetic forces acting on moving charges (see art 2.6-2).

Case III. ***System involving non-potential forces :***

Case I & II both include potential force component derivable either from a veloc[ity] independent ordinary potential V or a velocity dependent generalised potential U. There are for[ces] which are not derivable from a potential function; for example Coriolis force $\mathbf{F} = 2m\,(\mathbf{v} \times \vec{\omega})$ and t[he] Lorentz force on a charged particle moving in a magnetic field $\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$. They are [...] examples of *gyroscopic forces. The power associated with these forces is zero i.e.* $\mathbf{F} \cdot \mathbf{v} = 0$.

If we denote nonpotential forces by Q'_j then Lagrange's equations of motion can be written

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=Q'_j. \qquad (j=1 \dots n)$$

where

$$Q'_j = Q'_j\,(q_1 \dots q_n, \dot{q}_1 \dots \dot{q}_n, t)$$

in general and $L = T - V$ or $L = T - U$ as the case may be.

We employ Newton's force laws to deduce Lagrange's equations of motion by this differential meth[od]. From this deduction, it atonce comes to the mind that such a formulation is a direct consequence of Newt[on's] law. Integral method (variational principle), however, maintains that Hamilton's principle, which foll[ows] directly from Lagrange's equations, (we have shown earlier the converse, since Hamilton's principle is bot[h] necessary and a sufficient condition for Lagrange's equations of motion) as a parallel fundamental postul[ate] of mechanics. Thus variational principle provides Lagrangian formulation a fundamental base, althou[gh] variation δq_j involved in variational principle are quite identical with virtual displacements of [...] co-ordinates encountered in differential method, since no variation of time is involved.

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