(b) **Electrical conductivity.** *Electrical conductivity is defined as the quantity of electricity (charge q) that flows in a unit time through a unit area of cross-section of the conductor per unit potential gradient (or electric field).*

If a charge q flows through a conductor of area of cross-section A in a time t under an electric field E, then

$$\text{Electrical conductivity } \sigma = \frac{q}{AtE}$$

According to free electron model, the electrons in a solid move freely. If e is the charge, m the mass of the electron and E the applied electric field, then acceleration produced in the electron

$$a = \frac{d^2x}{dt^2} = \frac{Ee}{m}$$

Integrating, we have

$$v = \frac{dx}{dt} = \frac{Eet}{m} + C$$

At $t = 0$, $\frac{dx}{dt} = 0$ $\quad\therefore\ C = 0$

$$\therefore \quad v = \frac{Eet}{m} \qquad \ldots(i)$$

and Energy of the electron $= \frac{1}{2}mv^2$

If T is the absolute temperature and k Boltzmann's constant, then

$$\text{Energy of the electron} = \frac{3}{2}kT$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{3}{2}kT$$

or
$$mv = \frac{3kT}{v} \qquad \ldots(ii)$$

If λ is the mean free path of the electron, then relaxation time between two successive collisions

$$T = \frac{\lambda}{v}$$

From Eq. (i) $v = \frac{Eet}{m}$

$\therefore$ Average velocity between two successive collisions

$$\bar{v} = \frac{1}{T}\int_0^T \frac{Ee}{m} t\, dt = \frac{Ee}{mT}\int_0^T t\, dt = \frac{Ee}{mT}\left[\frac{t^2}{2}\right]_0^T = \frac{1}{2}\frac{Ee}{mT}\cdot T^2$$

or $$\bar{v} = \frac{1}{2}\frac{EeT}{m}$$

Substituting $T = \frac{\lambda}{v}$, we get

$$\bar{v} = \frac{1}{2}\frac{Ee\lambda}{mv} \quad ...(iii)$$

Substituting the value of $mv = \frac{3kT}{v}$ from Eq. (ii) in Eq. (iii), we have

$$\bar{v} = \frac{1}{2}\frac{Ee\lambda v}{3kT} = \frac{Ee\lambda v}{6kT}$$

If n is the number density of electrons in the conductor, then current density

$$J = ne\bar{v} = \frac{ne^2 E\lambda v}{6kT} \quad ...(iv)$$

If q is the quantity of charge flowing through the conductor of cross-sectional area A in time t, then

$$q = \sigma AEt$$

or $$J = \frac{q}{At} = \sigma E \quad ...(v)$$

Comparing Eqs. (iv) and (v), we get

$$\sigma = \frac{ne^2\lambda v}{6kT} \quad ...(vi)$$

Ohm's law. From Eq. (iv), we get

$$J = \frac{ne^2 E\lambda v}{6kT}$$

or $$\vec{J} = \sigma \vec{E} \text{ where } \sigma = \frac{ne^2\lambda v}{6kT}$$

which is vector form of Ohm's law.

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