4.1 DE-BROGLIE HYPOTHESIS FOR MATTER WAVES

According to quantum theory, radiation of frequency ν consists of quanta or photons, each of energy $E = h\nu$ where h is Planck's constant the value of which is $= 6.62 \times 10^{-34}$ Joule sec.

The equivalent mass of the photon $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$

Since the speed of the photon in free space is c, the equivalent momentum of the photon

$p = mc = \frac{E}{c^2} c = \frac{h\nu}{c} = \frac{h}{\lambda}$ where λ is the wavelength of the radiation of frequency ν.

By analogy with this, de-Broglie suggested that a moving particle is associated with a wave. The frequency of the wave is taken to be $\nu = \frac{E}{h} = \frac{mc^2}{h}$ where m is the mass of the particle. The wavelength of the wave $\lambda = \frac{h}{p} = \frac{h}{mv}$ where v is the velocity of the particle.

Thus, a particle of mass m moving with a velocity v has an associated wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad ...(4.1)$$

This is known as de-Broglie *wave equation* and λ is called de-Broglie *wavelength*. The associated wave is termed *matter*, *guide*, *pilot* or de-Broglie wave.

de-Broglie wave velocity or phase velocity of the wave is given by

$$v_p = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v}$$

Since the particle velocity v, must be less than the velocity of light c, de-Broglie waves travel faster than velocity of light.

Like other waves the de-Broglie wave can also be represented by a function $\Psi(\vec{r}, t)$ *i.e.* a function of position $\vec{r}$ and time t. It is known as *wave function* representing *matter wave* guiding the particle. The wave function can be positive, negative or a complex quantity. It has an amplitude or modulus and a phase like other complex functions. The wave vector $\vec{k}$ for de-Broglie wave is given by

$$|\vec{k}| = k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{p}{h/2\pi} = \frac{p}{\hbar}$$

where

$$\hbar = \frac{h}{2\pi}$$

4.1.1 de-Broglie wavelength in terms of energy and temperature. de-Broglie wavelength according to relation (4.1) is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If E is the kinetic energy of the moving particle, then

$$E = \frac{1}{2} mv^2 = \frac{1}{2}\frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

or

$$p = \sqrt{2mE} \quad \text{...(4.2)}$$

Hence de-Broglie wavelength $\quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$ ...(4.3)

According to kinetic theory of gases, the average kinetic energey of the material particle is given by

$$E = \frac{1}{2} mv^2 = \frac{3}{2} kT$$

where k is Boltzmann's constant = 1.38×10^{-23} JK^{-1} and T is the absolute temperature.

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}} \quad \text{...(4.4)}$$

Thus, wavelength associated with a moving particle is proportional to $1/\sqrt{T}$.

EC-21

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2. Subject — PHYSICS
3. Paper —
4. TDC — III
5. Topic — De-Broglie Hypothesis of matter wa[illegible]
6. Date — 14-01-2022

EC-22

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