5.1 HEISENBERG'S UNCERTAINTY PRINCIPLE

The de-Broglie wavelength associated with a moving particle travelling with a uniform velocity v is given by

$$\lambda = \frac{h}{mv}$$

This is a monochromatic wave of *infinite* extent. The phase velocity v_p of such a monochromatic de-Broglie wave is given by

$$v_p = v\lambda = \frac{hv}{h/\lambda} = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

and is thus always greater than the velocity of light in vacuum. It is, therefore, theoretically impossible for a monochromatic de-Broglie wave train to transport a particle or carry energy. The phase velocity, merely represents the rate at which a given phase of a mono-chromatic wave train advances and is purely a mathematical concept.

Further, the stability of a material particle like an electron demands that it should be concentrated over a small region of space at any given instant of time. Thus, the mass and charge of a particle like the electron are localised entities whereas the de-Broglie wave with which we represent the moving particle is of infinite extent by virtue of its being monochromatic.

By taking a group of waves of nearly equal wavelengths centered round the de-Broglie wavelength $\lambda = \frac{h}{p}$ we obtain a *wave packet* the amplitude of which is different from zero only in a certain small region of space having dimensions of the particle. This small region of space is associated with the position of the particle and this wave packet travels with the velocity known as group velocity v_g which is equal to the particle velocity v.

Hence, instead of associating a single monochromatic de-Broglie wave with a particle we associate with it a wave packet consisting of a group of waves of nearly equal amplitude centred round de-Broglie wavelength $\lambda = \frac{h}{p}$ of the particle. The wave packet has the dimensions of the localised particle and travels with the same velocity as the particle.

The association of a group of waves with a moving particle means that the position of the particle at any instant of time cannot be specified with any desired degree of accuracy. All that we can say is that the particle is somewhere within the group of waves i.e. within a small region of space. Thus, a certain inaccuracy will always creep in ascertaining the position of a moving particle like the electron, proton etc. With a similar reasoning it can be argued that a certain amount of inaccuracy will creep in determining the velocity and hence the momentum of the particle.

Heisenberg's uncertainty principle states that in any simultaneous determination of the position and momentum of a particle, the product of the uncertainties is equal to, or greater than Planck's constant h.

$$\Delta x . \Delta p_x \geq h$$

i.e.

for motion along X-axis, where Δx is the fundamental error or uncertainty in the measurement of the position and Δp_x the fundamental error or uncertainty in the measurement of momentum (velocity).

A more accurate calculation gives

$$\Delta p_x \; \Delta x \geq \frac{1}{2} \frac{h}{2\pi} = \frac{1}{2} \hbar$$

According to classical ideas, it is possible for a particle to occupy a fixed position and have a definite momentum and we can predict exactly its position and momentum at any time later. According to uncertainty principle, it is not possible to determine accurately the simultaneous values of position and momentum of a particle at any time. If it is desired to reduce the error, in the determination of position *i.e.*, to reduce Δx, then this can only be done at the expense of accuracy in determining the momentum and the error involved in Δp will increase.

It should be born in mind that, uncertainties are not due to inperfections of the measuring instruments or way of measurements. These are inherent due to wave nature of the particle.

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1. Name — Dr. Ajmyt Singh
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