5.8 APPLICATIONS OF UNCERTAINTY PRINCIPLE

5.8.1 Non-existence of Electrons in Nucleus

If a particular type of particle is to exist within the nucleus then the uncertainty in its p in the nucleus should not exceed the radius of the nucleus. The radius of a typical atomic n is of the order of 10^{-14} m. Hence if the electron is to exist within the nucleus the uncertain position is given by

$$\Delta x \geq 10^{-14} \text{ m}$$

According to Heisenberg's uncertainty principle,

$\Delta x . \Delta p_x = \frac{\hbar}{2}$ where Δp_x is the uncertainty in the x-component of momentum or the mom

itself and $\hbar = \frac{h}{2\pi}$

∴ Uncertainty in the momentum of the electron if it is to exist in the nucleus

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{6.6 \times 10^{-34}}{2 \times 2\pi \times 10^{-14}} = 5.3 \times 10^{-21} \text{ kg ms}^{-1}$$

This means that the total momentum of the *electron* in the nucleus must be at least of the order of 5.3×10^{-21} kg ms^{-1}.

The rest mass of the electron $= 9.1 \times 10^{-31}$ kg.

Hence the above value of the momentum is *relativistic* as a non-relativistic relation for momentum gives a value of $v > c$.

According to the theory of relativity, the total relativistic energy of the electron

$$E = [m_0^2 c^4 + p^2 c^2]^{1/2}$$

$$= [(9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4 + (5.3 \times 10^{-21})^2 \times (3 \times 10^8)^2]^{1/2}$$

$$= 3 \times 10^8 \, [9.1 \times 9.1 \times 9 \times 10^{-46} + 5.3 \times 5.3 \times 10^{-42}]^{1/2}$$

$$= 16 \times 10^{-13} \text{ Joule} = \frac{16 \times 10^{-13}}{1.6 \times 10^{-19}} = 10 \text{ MeV}.$$

Thus, the kinetic energy of a free electron confined within a nucleus should have a minimum value of 10 MeV. The rest mass energy of an electron is only about 0.5 MeV and experimentally, the highest value of the kinetic energy of a β-particle emitted spontaneously by a nucleus is found to be about 4 MeV. Since this is much less than the calculated value of the minimum kinetic energy which the electron, if it exists inside the nucleus, should possess, we come to the conclusion that electrons cannot be constituents of the atomic nucleus.

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