6.7 TIME INDEPENDENT SCHRÖDINGER'S WAVE EQUATION

The time dependent, three dimensional Schrödinger's wave equation is given by

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t} \qquad ...(i)$$

and explicitly in terms of $\vec{r}$ and t, we have

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V\Psi(\vec{r},t) = i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} \qquad ...(ii)$$

The one dimensional Schrödinger's wave equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t} \qquad ...(iii)$$

and explicitly in terms of x and t, we have

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad ...(iv)$$

When the potential energy V does not depend explicitly on time and is a function of x only and the total energy E is constant, the wave function $\Psi(x, t)$ can be written as the product of two separate functions $\psi(x)$ a function only of x and $f(t)$ a function only of t.

$\therefore \quad \Psi(x, t) = \psi(x) f(t)$

The above relation is briefly written as $\Psi = \psi f$

Hence $\quad \frac{\partial\Psi}{\partial x} = f\frac{\partial\psi}{\partial x}$ and $\frac{\partial^2\Psi}{\partial x^2} = f\frac{\partial^2\psi}{\partial x^2}$ and $\frac{\partial\Psi}{\partial t} = \psi\frac{\partial f}{\partial t}$

Now $\quad \Psi = Ae^{\frac{i}{\hbar}(xp_x - Et)}$

$$\therefore \quad \frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}EAe^{\frac{i}{\hbar}(xp_x - Et)} = -\frac{i}{\hbar}E\Psi = \frac{-i}{\hbar}E\psi f \qquad ...(v)$$

Substituting in equation (*iii*), we have

$$-\frac{\hbar^2}{2m} f \frac{\partial^2 \psi}{\partial x^2} + Vf\psi = i\hbar\left(\frac{-i}{\hbar}\right)E\psi f$$

Dividing both sides by *f*, we have

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

As ψ is a function of *x* only it may be stated explicitly

as

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi(x) = 0$$

Further ψ is a function of *only one* variable *x*, we can replace *partial derivative* by derivative $\frac{d\psi}{dx}$ and $\frac{d^2\psi}{dx^2}$.

We can, therefore, write equation (*viii*) as

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

This is *one dimensional, time independent* Schrödinger's equation.

Three dimensional time independent equation. For motion of the particle in three dimensions ψ is a function of *x, y, z* or $\vec{r}$. In such a case the *time independent* or *stationary state* Schrödinger's quantum mechanical wave equation is given by

$$\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

or

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

More explicitly

$$\nabla^2 \psi(\vec{r}) + \frac{2m}{\hbar^2}(E - V)\psi(\vec{r}) = 0$$

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