6.17 OPERATOR

An operator is a mathematical rule or prescription. Mathematical operations in algebra, calculus like adding, subtracting, multiplying, dividing, finding the square root, differentiating with respect to a variable, or integration are represented by characteristic symbols like, $+, -, \times, \div, \sqrt{}$ and $\int f dx$ can be considered as operators.

Thus if A is an operator generally represented as $\hat{A}$ and stands for the operation $\frac{\partial}{\partial x}$, then

$$\hat{A}x^3 = \frac{\partial}{\partial x}(x^3) = 3x^2$$

and
$$4x^2\,\hat{A}x^3 = 4x^2\frac{\partial}{\partial x}(x^3) = 12x^4.$$

Commuting and non-commuting operators. Consider two operators $\hat{A}$ and $\hat{B}$ where $\hat{A}$ stands for $\frac{\partial}{\partial x}$ and $\hat{B}$ stands for the operation $x^2 \times$, then if the two operators $\hat{A}$ and $\hat{B}$ operate together on a function say y which is a function of variable x, then

$$\hat{A}\hat{B}\,y = \frac{\partial}{\partial x}(x^2\times)\,y = \frac{\partial}{\partial x}x^2 y = 2xy + x^2\frac{\partial y}{\partial x}$$

On the other hand

$$\hat{B}\hat{A}\,y = (x^2\times)\frac{\partial}{\partial x}\,y = x^2\frac{\partial y}{\partial x}$$

Thus if there are two operators applied together $\hat{A}\;\hat{B}$ means, first apply $\hat{B}$, then $\hat{A}$ and $\hat{B}\hat{A}$ means first apply $\hat{A}$ then $\hat{B}$. From the above we find that $\hat{A}\hat{B} \neq \hat{B}\hat{A}$ because $\hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$.

Now consider two operators $\hat{A}$ and $\hat{B}$ where $\hat{A}$ represents $\frac{\partial}{\partial x}$ and $\hat{B}$ represents $a \times$; a a constant, then

$$\hat{A}\hat{B}\,y = \frac{\partial}{\partial x}(a\times)y = \frac{\partial}{\partial x}ay = a\frac{\partial y}{\partial x}$$

and
$$\hat{B}\hat{A}\,y = (a\times)\frac{\partial}{\partial x}\,y = a\frac{\partial y}{\partial x}$$

Thus
$$\hat{A}\hat{B} = \hat{B}\hat{A} \text{ and } \hat{A}\hat{B} - \hat{B}\hat{A} = 0.$$

Two operators $\hat{A}$ and $\hat{B}$ for which $\hat{A}\hat{B} = \hat{B}\hat{A}$ or for which $\hat{A}\hat{B} - \hat{B}\hat{A} = 0$ are said to be *commuting*.

Commutator. The commutator of two operators $\hat{A}$ and $\hat{B}$ denoted as $[\hat{A}, \hat{B}]$ is defined as $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$.

Linear operator. An operator $\hat{A}$ is said to be *linear* if it satisfies the following two equations

$$\hat{A}(\alpha f) = \alpha \hat{A} f \qquad ...(i)$$

$$\hat{A}(f + g) = \hat{A} f + \hat{A} g \qquad ...(ii)$$

where α is a constant

Linear momentum operator see article 6.19 (*iii*) **Energy operator** see article 6.19 (iv) and (v).

Hermitian operator. An operator $\hat{A}$ is called *Hermitian* if it satisfies the equation

$$\int_{-\infty}^{+\infty} f^*(\hat{A} g)\, dx = \int_{-\infty}^{+\infty} (\hat{A} f)^* g\, dx$$

where $f(x)$ and $g(x)$ are any two well behaved functions of x.

Hermitian operator may also be written as

$$\int_{-\infty}^{\infty} f^* \hat{A} g\, dx = \int_{-\infty}^{\infty} g \hat{A}^* f^* dx.$$

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