6.25 ORTHOGONAL WAVE FUNCTIONS

The two wave functions $\psi_m(x)$ and $\psi_n(x)$ are said to be orthonormal or orthogonal when

$$\int_{\text{Available region}} \psi_m^*(x)\, \psi_n(x)\, dx = 0$$

Wave functions of two different states are orthogonal. The wave functions for two different states m and n of a free particle in a box are given by

$$\psi_m(x) = \left(\frac{2}{l}\right)^{1/2} \sin \frac{m\pi}{l} x \text{ and } \psi_n(x) = \left(\frac{2}{l}\right)^{1/2} \sin \frac{n\pi}{l} x$$

$$\therefore \int_0^l \psi_m^*(x)\, \psi_n(x)\, dx = \frac{2}{l} \int_0^l \sin \frac{m\pi}{l} x \sin \frac{n\pi}{l} x$$

$$= \frac{2}{l}\frac{1}{2} \int_0^l \left[\frac{\cos(m-n)\pi x}{l} - \frac{\cos(m+n)\pi x}{l}\right] dx$$

$$= \frac{1}{l}\left[\frac{\sin(m-n)\pi x/l}{(m-n)\pi/l}\right]_0^l - \left[\frac{\sin(m+n)\pi x/l}{(m+n)\pi/l}\right]_0^l = 0 \qquad ...(6.40)$$

since both the functions within the brackets are each equal to zero.

Hence the wave functions for two different states are orthogonal or orthonormal.

Two lowest states. For the two lowest states $n = 1$, $m = 2$ and the result holds good.

EC-(32)

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