EC-116

Subject : PHYSICS
Topic : Mean Energy of Planck's Oscillations.
Class : TDC-I & Paper-II
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...erage energy of Planck's oscillator: If N is the total number of Planck's resonators and E ...al energy, then average energy per Planck's oscillator is given by

$$\bar{\varepsilon} = \frac{E}{N} \qquad ...(vii)$$

...cording to Maxwell's law of molecular motion, if ε is a energies $0, \varepsilon, 2\varepsilon, 3\varepsilon, ..., r\varepsilon, ...$ ...the ratio $1 : e^{-\varepsilon/kT} : e^{-2\varepsilon/kT} : e^{-3\varepsilon/kT} : .... : e^{-r\varepsilon/kT}$ .... etc.

...If N_0 is the number of resonators having energy zero, then the number of resonators N_1 ...g energy ε will be $N_0\, e^{-\varepsilon/kT}$, the number of resonators N_2 having energy 2ε will be N_0 ...and in general, the number of resonators N_r, having energy $r\varepsilon$ will $N_0\, e^{-r\varepsilon/kT}$ and so on.

$$N = N_0 + N_1 + N_2 + ... + N_r + ...$$

$$= N_0 + N_0 e^{-\varepsilon/kT} + N_0\, e^{-2\varepsilon/kT} + ... + N_0\, e^{-r\varepsilon/kT} + ...$$

$$= N_0\, [1 + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} + ... + e^{-r\varepsilon/kT} + ...]$$

Putting $e^{-\varepsilon/kT} = y$, we have

$$N = N_0\, [1 + y + y^2 + ... + y^r + ...]$$

or

$$N = \frac{N_0}{1-y}$$

$$e^{-\frac{\varepsilon}{kT}\times 2} = \gamma\gamma$$

$$e^{-\frac{2\varepsilon}{kT}} = e^{-\frac{\varepsilon}{kT}} \times \gamma = \gamma$$

The total energy of Planck's resonator will be

$$E = o \times N_0 + \varepsilon \times N_1 + 2\varepsilon \times N_2 + ... + r\varepsilon \times N_r + ...$$

$$= o \times \varepsilon\, N_0\, e^{-\varepsilon/kT} + 2\varepsilon\, N_0\, e^{-2\varepsilon/kT} + ... + r\varepsilon\, N_0\, e^{-r\varepsilon/kT} + ...$$

$$= N_0\, \varepsilon\, [e^{-\varepsilon/kT} + 2\varepsilon\, e^{-2\varepsilon/kT} + re^{-r\varepsilon/kT} ....]$$

$$= N_0 \varepsilon \frac{y}{(1-y)^2}$$

Therefore, the average energy of a resonator will be

$$\bar{\varepsilon} = \frac{E}{N} = \frac{N_0 \varepsilon . \frac{y}{(1-y)^2}}{\frac{N_0}{1-y}}$$

$$= \frac{\varepsilon y}{1-y} = \frac{\varepsilon e^{-\varepsilon/kT}}{1-e^{-\varepsilon/kT}} = \frac{\varepsilon}{e^{\varepsilon/kT}-1}$$

According to Planck's hypothesis of quantum theory, $\varepsilon = h\nu$, therefore, the average ene
Planck's oscillator is given by

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/kT}-1}$$

Therefore, the *energy density* belonging to the range $d\nu$ can be obtained by multiplyin average energy of Planck's oscillator by the number of oscillators per unit volume, in this frequ range ν and $(\nu + d\nu)$.

i.e.,
$$E_\nu \, d\nu = \left(\frac{8\pi\nu^2}{c^3} d\nu\right)\left(\frac{h\nu}{e^{h\nu/kT}-1}\right)$$

or
$$= \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT}-1} d\nu \qquad ...(8.14)$$

where $E_\nu \, d\nu$ is energy density (*i.e.,* total energy per unit volume) belonging to the range $d\nu$
Equation (8.14) is called *Planck's radiation law.*

The energy density $E_\lambda \, d\lambda$ belonging to range $d\lambda$ can be obtained by using the relation $\nu = \frac{c}{\lambda}$ a

hence $|d\nu| = \left|-\frac{c}{\lambda^2} d\lambda\right|$ we get

$$E_\lambda \, d\lambda = \frac{8\pi h}{c^3}\left(\frac{c^3}{\lambda^3}\right) \cdot \frac{1}{e^{hc/\lambda kT}-1} \cdot \left|-\frac{c}{\lambda^2} d\lambda\right|$$

$$= \frac{8\pi hc}{\lambda^5} \cdot \frac{d\lambda}{e^{hc/\lambda kT}-1}$$

This is *Planck's radiation law* in terms of wavelength.