Subject : PHYSICS

E-Content No. : 4

Topic : Carnot's Engine & Carnot's Theorem

Class : B.Sc. (Hons) – Part-I, Paper-II

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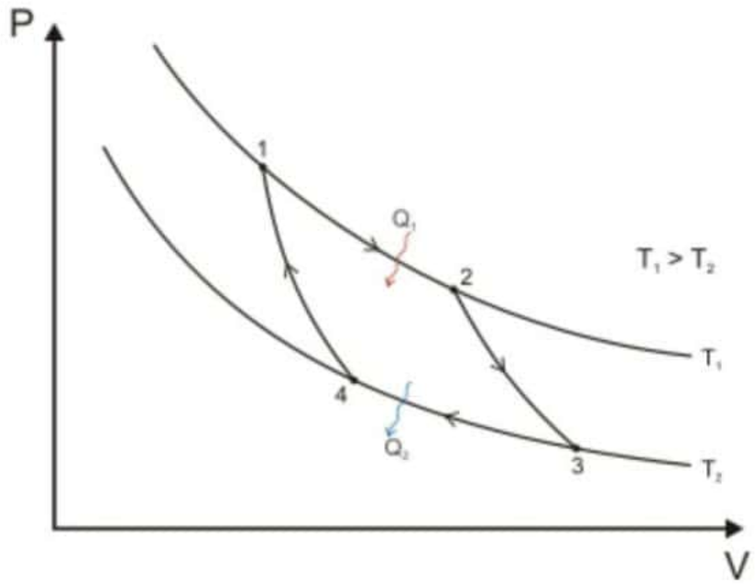
x ——— x ——— x ——— x ——— x ——→

1.2.2 Carnot engine

Carnot engine is an engine consisting of two isothermal paths (i.e. $\Delta T = 0$) operating at two different temperatures and two adiabatic paths (i.e. $\Delta Q = 0$) connected in a cyclic transformation.



The efficiency of engines is defined as

$$\eta = \frac{W}{Q_1}, \tag{1.7}$$

and for a Carnot engine it is given by

$$\eta = 1 - \frac{Q_2}{Q_1}. \tag{1.8}$$

Carnot's theorem: a) All irreversible engines operating between temperatures T_1 and $T_2 < T_1$ are less efficient than a Carnot engine operating between the same temperatures.

b) All reversible engines operating between temperatures T_1 and $T_2 < T_1$ are equally efficient as a Carnot engine operating between the same temperatures.

Proof: a) Combine an arbitrary heat engine whose efficiency is η with a reversed Carnot engine whose efficiency is η' so that there is no work done by the combined system. If $\eta = \eta'$ then the process does nothing in conflict with irreversibility assumption. If $\eta > \eta$' then the process is in the conflict

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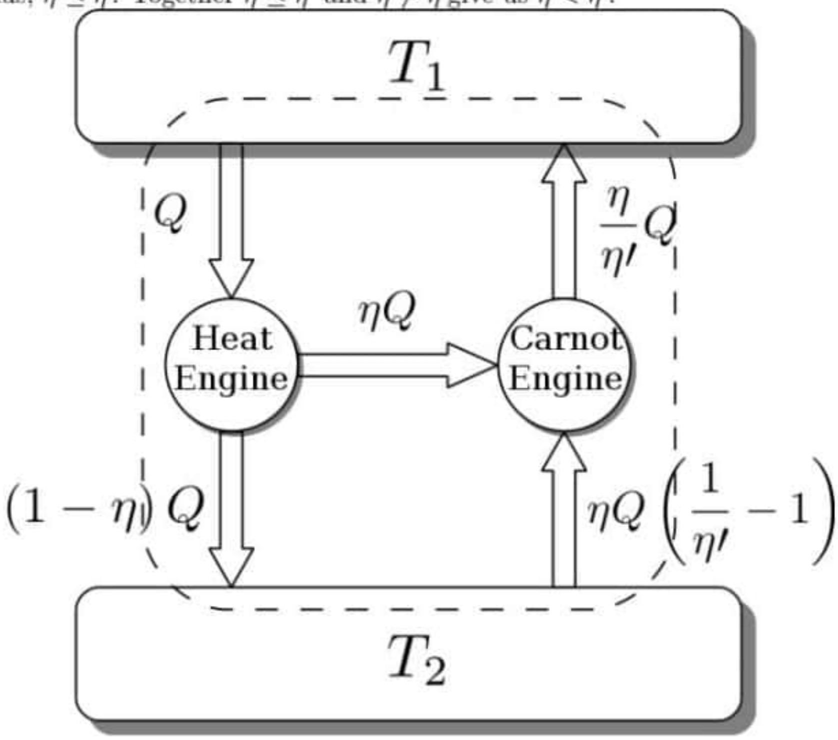
with Clausius statement of the Second Law, i.e.

$$\Delta Q = \frac{\eta}{\eta'}Q - Q > 0.$$

Thus, $\eta \leq \eta'$. Together $\eta \leq \eta'$ and $\eta \neq \eta'$give us $\eta < \eta'$.



b) We have already proved that $\eta \leq \eta'$ regardless of reversibility. Now we can reverse the process by combining a reversed heat engine with a Carnot engine which leads to conclusion $\eta' \leq \eta$. Together $\eta \leq \eta'$ and $\eta' \leq \eta$ give us $\eta = \eta'$.

According to Carnot's theorem the efficiency of reversible process between any two temperatures is a universal number, i.e. $\eta(T_1, T_2)$. This allows us to define not only relative, but also absolute temperature scale. Consider three Carnot cycles 1-2, 2-3 and 1-3 operating between different temperatures $T_1$and T_2, T_2 and T_3, T_1 and T_3 respectively, where without loss of generality we assume that $T_1 > T_2 > T_3$. We can now construct a combine cycle such that the heat Q_2 rejected by 1-2 is absorbed by 2-3 which is a reversible and thus a cycle 1-3 by Carnot's theorem. Then the heat absorbed by reservoir at T_3 must satisfy both

$$Q_3 = Q_1 - W_{13} = Q_1(1 - \eta(T_1, T_3))$$

$$Q_3 = Q_2 - W_{23} = Q_2(1-\eta(T_2, T_3)) = (Q_1 - W_{12})(1-\eta(T_2, T_3)) = Q_1(1-\eta(T_1, T_2))(1-\eta(T_2, T_3)).$$

Therefore

$$(1 - \eta(T_1, T_3)) = (1 - \eta(T_1, T_2))(1 - \eta(T_2, T_3))$$

and

$$1 - \eta(T_1, T_2) = \frac{f(T_2)}{f(T_1)} \tag{1.9}$$

for an arbitrary function $f(T)$ which is by convention is set to be a linear function, i.e.

$$\eta = 1 - \frac{T_2}{T_1} \tag{1.10}$$

and from the definition of efficiency

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}. \tag{1.11}$$

1.2.3 Entropy

The Second Law suggests a new thermodynamical quantity, called entropy and usually denoted by S. It is conveniently introduced using Clausius's theorem.

Clausius's Theorem: For any cyclic transformation

$$\oint \frac{dQ}{T} \leq 0. \tag{1.12}$$

The equality holds for cyclic transformations.

Proof: Subdivide the cycle into infinitesimal transformations where the temperature T remains roughly constant. During each transformation the system receives dQ of heat and does dW of work. Arrange a series of Carnot cycles operating between temperatures T and T_R where is the temperature of an arbitrary reservoir. The whole purpose of each Carnot cycle is to deliver to the system dQ of heat and to do dW of work. Then according to the absolute definition of temperature

$$\frac{dW - dQ}{dQ} = \frac{T_R}{T}. \tag{1.13}$$

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By integrating of entire cycle the total heat received by the system is zero $\oint dQ = 0$ and by the Kelvin's statement of the Second Law the total work must be non-positive $\oint dW \leq 0$. Thus,

$$\oint (dW - dQ) = T_R \oint \frac{dQ}{T} \leq 0 \Rightarrow \oint \frac{dQ}{T} \leq 0 \qquad (1.14)$$

since $T_R > 0$.

For a reversible cycle we can run the process in opposite direction to show that $\oint \frac{dQ}{T} \geq 0$ which together with $\oint \frac{dQ}{T} \leq 0$ implies $\oint \frac{dQ}{T} = 0$.

An immediate consequence of the Clausius's Theorem is that for reversible transformations between two states A and B the integral $\int_A^B \frac{dQ}{T}$ does not depend on the path. Indeed if we take two distinct reversible paths parametrizes by dQ and dQ' then together they would form a reversible cycle, i.e.

$$\int_A^B \frac{dQ}{T} + \int_B^A \frac{dQ'}{T} = 0 \qquad (1.15)$$

and thus,

$$\int_A^B \frac{dQ}{T} = \int_A^B \frac{dQ'}{T}. \qquad (1.16)$$

This suggests that for a reversible transformation we can define and exact differential

$$dS \equiv \frac{dQ}{T}. \qquad (1.17)$$

whose integral defines relative entropy up to an arbitrary constant of integration. This also produces a new pair of conjugate thermodynamic variables generalized force T and generalized displacement S, i.e.

$$dU = TdS - PdV. \qquad (1.18)$$

Although the entropy was defined using only reversible processes (as a reference), the Clausius's Theorem implies that for all systems

$$S(B) - S(A) \geq \int_A^B \frac{dQ}{T} \qquad (1.19)$$

and for thermally isolated systems (i.e. $dQ = 0$)

$$S(B) - S(A) \geq 0 \qquad (1.20)$$

where the equalities hold for only reversible transformations. This shows that in a thermal equilibrium (when the state of the system does not change) the system must be in a state of maximal entropy.

Second Law in symbols:

$$\frac{dS}{dt} \geq 0 \tag{1.21}$$

Not all of the processes we observe in nature are reversible. Many processes are irreversible, which is a rather surprising fact given that the fundamental laws of physics (as we know them) are usually time symmetric. Then, why does the nature always picks some initial conditions and not other? Why don't we see a thermal state? Why is there an asymmetry between past and future?

Question To Go: Why do we see an arrow of time?