1.5 APPLICATION OF FERMAT'S PRINCIPAL

On the basis of Fermat's principle you can derive laws of reflection and refraction.

1.5.1. Laws of Reflection

When light ray falls on a smooth polished surface separating two media, it comes back in the same medium, the phenomenon is called reflection and the boundary is called reflecting surface. The light obeys following two laws of reflection.

First law:

The incident ray, reflected ray and the normal to the surface at the point of incidence all lie in one plane. You can prove this law in the following way.

Let the plane ABCD be normal to the plane mirror shown in figure 1.1. P is point object imaged by mirror as P'. Consider a point M' on the plane mirror; but not on plane ABCD. Let a ray PM' be reflected as $M'P'$. Draw a normal $M'M$ on plane ABCD. Point M is the foot of the normal on ABCD.

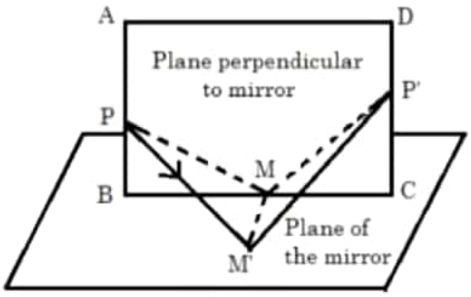

Fig. 1.1

Now, PMM' and $P'MM'$ are right angle triangles. PM' and $P'M'$ are respective hypotenuse. Therefore, we have

$$PM' > PM \text{ and } P'M' > P'M$$

But Fermat's principle demands that the path followed must be the shortest, i.e., the light would not travel along $PM'P'$. As we shift M' towards M the path of light ray becomes shorter. It is seen that the shortest possible path is $PM\,P'$, where the point of incidence M lies on plane ABCD. PM and MP' are the incident and reflected rays. This proves the first law of reflection.

Second law:

For a smooth surface, the angle of incidence is equal to the angle of reflection. You can prove the second law as follows.

Assuming DD' is a reflecting plane shown in figure 1.2. Object P is imaged as P' and M is the point of incident. The normal to the plane at this point is MN and is shown by dotted line.

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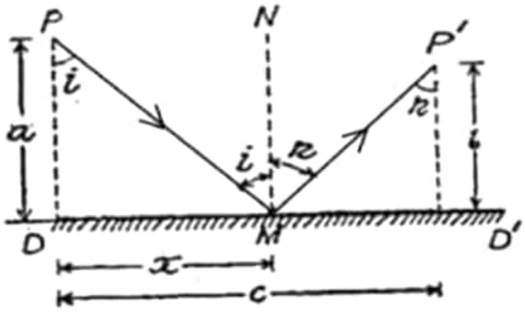

Fig. 1.2

PM and *MP'* are the incidence and reflected rays. Let *I* and *r* are the angles of incidence and reflection respectively. Let us suppose distances

$$PD = a,\ P'D' = b,\ DM = x,\ DD' = c.$$

The ray of light travels in air from *P* to *P'*. Let the path *PMP'* be *s*, then the total distance covered by light ray be

$$s = P\,MP' = PM + MP'$$

$$= (PD^2 + DM^2) + (D'M^2 + D'P'^2)$$

$$= (a^2 + x^2) + \{(c - x)^2 + b^2\} \qquad \ldots\ldots\ (1.8)$$

It is evident that the path from P to P' remains the same even if the point of incidence M shifts. Shifting of M changes *x* only. According to Fermat's principle the path *PMP'* must be either minimum or maximum. It means that the differential coefficient of *s* with respect to *x* must be zero, i.e.

$$\frac{ds}{dx} = \frac{1}{2}\frac{2x}{\sqrt{(a^2+x^2)}} - \frac{1}{2}\frac{2\,(c-x)}{\sqrt{\{(c-x)^2+b^2}} = 0$$

or

$$\frac{x}{\sqrt{(a^2+x^2)}} = \frac{(c-x)}{\sqrt{\{(c-x)^2+b^2\}}}$$

From figure 1.2, we have,

$$\frac{x}{\sqrt{(a^2+x^2)}} = \sin i,\ (c-x)/\sqrt{\{(c-x)2 + b2\}} = \sin r$$

∴

$$\sin i = \sin r \qquad \ldots\ldots\ (1.9)$$

or

$$i = r$$

Hence you can see that the second law of reflection is derived from Fermat's principle. Further the second differential co-efficient of s, i.e., $\frac{d^2s}{dx^2}$ comes out to be positive, which proves that the path is minimum (or path of least time).

1.5.2. Laws of Refraction

When a ray of light passes from one homogenous medium to another, the phenomenon of bending of light ray towards or away from the normal is called refraction. Again there are following two laws of refraction.

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