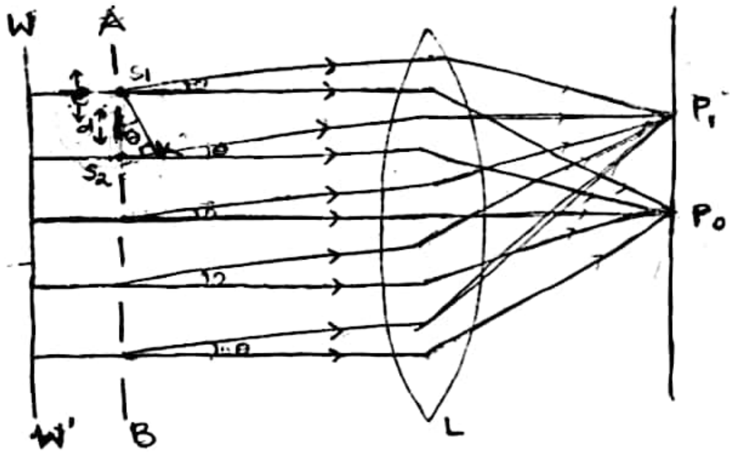
1.9 REFERENCES

1. A text Book of Optics – Brijlal, S.Chand Publishing, N.Delhi
2. Optics-Satya Prakash, Pragati Prakashan, Meerut
3. Introductory University Optics, PHI Learning, New Delhi, 1998.

Diffraction due to plane transmission grating /Fraunhoffer diffraction due to N-parallel slit:

Let us consider a plane wave front coming from an infinite distance is allowed to incident on a convex lens “L” which is placed at its focal length. The rays of light which are allowed to incident normally on the lens are converged to a point "P_o" forming central principal maxima having high intensity and the rays of light which are diffracted through an angle are "θ" are converge to a point "P_1" forming a minima having less intensity as compared to central principal maxima. Again those rays of light which are diffracted through an angle "θ" are undergoes a path difference and hence a phase difference producing diffraction.



Let AB- be the transverse section of the plane transmission grating

ww'- be a plane wave front coming from infinite distance

e = width of the slit

d = width of the opacity

(e+d) = grating element of the grating

N = be the no. of rulings present in the grating

Now the path difference between the deviated light rays is

$$S_2K = S_1S_2 Sin\theta = (e+d)Sin\theta$$

Therefore, Phase difference = $\frac{2\pi}{\lambda}$ x $S_2K = \frac{2\pi}{\lambda}(e+d)Sin\theta = 2\beta$ (say)

where $\beta = \frac{\pi}{\lambda}(e+d)Sin\theta$

Now the resultant amplitude due to superposition of "N" no .of waves coming from "N" parallel slit is given as

$$R = A\frac{Sin\alpha}{\alpha}\frac{SinN\beta}{Sin\beta}$$

and intensity is given as $I\alpha R^2 \Rightarrow I = KR^2 = KA^2\frac{Sin^2\alpha}{\alpha^2}\frac{Sin^2N\beta}{Sin^2\beta} = I_0\frac{Sin^2\alpha}{\alpha^2}\frac{Sin^2N\beta}{Sin^2\beta}$

where $I_0\frac{Sin^2\alpha}{\alpha^2}$ =this is contributed due to diffraction at single slit

and $\frac{Sin^2N\beta}{Sin^2\beta}$ = this is contributed due to interference at " N" parallel slit

Position for central principal maxima /condition for central principal maxima:

The principal maxima will be obtained when

$$Sin\beta = o = Sin(\pm m\pi)$$
$$\Rightarrow \beta = \pm m\pi$$
$$\Rightarrow \frac{\pi}{\lambda}(e+d)Sin\theta = \pm m\pi$$
$$\Rightarrow (e+d)Sin\theta = \pm m\lambda$$

where $m = 0,1,2,3......$ .This is called grating equation or condition for central principal maxima.

Position for minima /condition for minima:

The minima will be obtained when

$$SinN\beta = o = Sin(\pm n\pi)$$

$$\Rightarrow N\beta = \pm n\pi$$

$$\Rightarrow N\frac{\pi}{\lambda}(e+d)Sin\theta = \pm n\pi$$

$$\Rightarrow N(e+d)Sin\theta = \pm n\lambda$$

Where n can take all the values except $n = 0, \pm N, \pm 2N, \pm 3N, ...$

This is the condition for minima due to diffraction at N-parallel slit.

Position/Condition for secondary maxima:

The maxima's occurring in between two consecutive secondary maxima is known as secondary maxima.

The positions for secondary maxima will be obtained as

$$\frac{dI}{d\alpha} = 0$$

$$\Rightarrow \frac{d}{d\alpha}\left[I_0 \frac{Sin^2\alpha}{\alpha^2}\frac{Sin^2 N\beta}{Sin^2\beta}\right] = 0$$

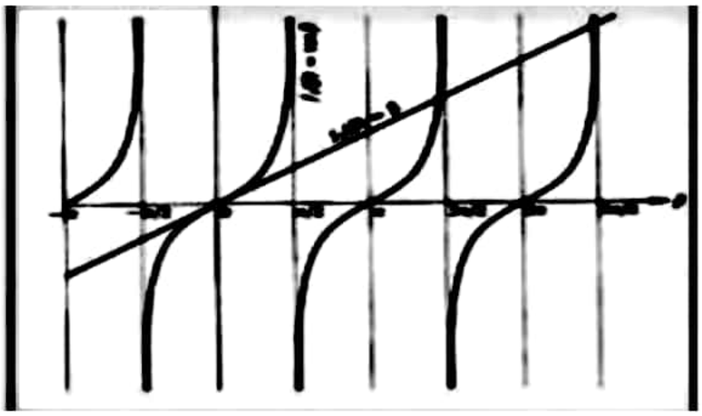
$$\Rightarrow 2I_0 \frac{Sin^2\alpha}{\alpha^2}\frac{Sin\ N\beta}{Sin\beta}\left[\frac{N\cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0$$

$$\Rightarrow \frac{N\cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta} = 0$$

$$\Rightarrow N\cos N\beta \sin\beta - \sin N\beta \cos\beta = 0 \Rightarrow N\cos N\beta \sin\beta = \sin N\beta \cos\beta$$

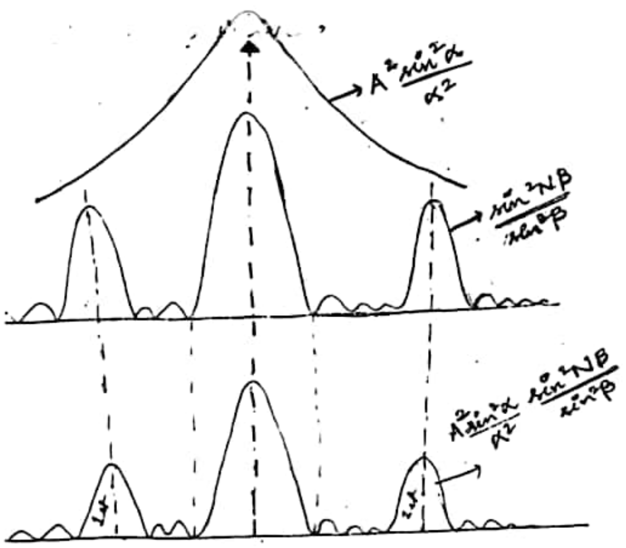
$$\Rightarrow N\tan N\beta = \tan N\beta$$

This is a transdectional equation. It can be solved by graphical method. Taking $y = \tan N\beta$ and $y = N\tan N\beta$, where the two plots are interests, this intersection points give the position for secondary maxima. Thus the secondary maxima's are obtained at $\beta = \frac{3\pi}{2}, \beta = \frac{5\pi}{2}, \beta = \frac{7\pi}{2}.........$



Intensity distribution curve:

The graph plotted between phase difference and intensity of the fringes is known as intensity distribution curve. The nature of the graph is as follows:



Characteristics of the spectral lines or grating spectra:

1.The spectra of different order are situated on either side of central principal maximum

2.Spectral lines are straight and sharp

3.The spectra lines are more dispersed as we go to the higher orders.

4.The central maxima is the brightest and the intensity decreases with the increase of the order of spectra.

Missing spectra or Absent spectra:

When the conditions for minima due to diffraction at single slit and condition for central principal maxima due to diffraction at N-parallel slit is satisfied simultaneously for a particular angle of diffraction then, certain order maxima are found to be absent or missed on the resulting diffraction pattern which are known as missing spectra or absent spectra.

Condition for Missing spectra:

We have,

The condition for central principal maxima due diffraction at N-parallel slit

$$(e+d)Sin\theta = \pm m\lambda$$

$$e\sin\theta = \pm n\lambda$$

$$\Rightarrow \frac{(e+d)Sin\theta}{e\sin\theta} = \frac{m\lambda}{n\lambda} = \frac{m}{n}$$

Special case:

1. If d = e, $\Rightarrow \frac{m}{n} = 2 \Rightarrow m = 2n$ where $n = 1,2,3,.....$

i.e second order or multiple of 2 order spectra will found to be missed or absent on the resulting diffraction pattern.

2. If $d = \frac{e}{2}, \Rightarrow \frac{m}{n} = \frac{3}{2} \Rightarrow m = 1.5n \approx 1$

i.e First order spectra will found to be missed or absent on the resulting diffraction pattern.

3. If $e = \frac{d}{2}, \Rightarrow \frac{m}{n} = 3 \Rightarrow m = 3n$

i.e Third order spectra or multiple of 3 spectra will found to be missed or absent on the resulting diffraction pattern.

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