Books for Study and Reference

1. Foundation of EMT – Third edition –John R. Reity, Frederick J. Milford and Robert W. Christy.
2. Electromagnetic theory – Prabir K. Basu and HrishikeshDhasmana.
3. Introduction to Electrodynamics– David J Griffiths.
4. Electromagnetic fields and waves– P.Lorrain and D.Corson.
5. Electrodynamics– B.P.Laud.

Scanned with CamScanner

6.19 BOUNDARY RELATIONS

1. Boundary conditions at the surface separating two substances

Let us examine, how the electric field changes at the boundary between two different media. The electric field change that occurs in going from one medium to another is determined by the two basic ideas in electrostatics, namely,

(i) the first is Gauss's law *i.e.*, $\oint_s \vec{D}.d\vec{S} = Q$ and

(ii) the second is that an electrostatics field is a conservative field, in other words, no work is done in transporting a change around a closed path in an electrostatics field *i.e.*,

$$\oint \vec{E}.d\vec{l} = 0$$

We shall apply Gauss's law to the cylindrical surface of height h and base area Δs as shown in Fig. 6.16. The cylindrical box is so constructed that it lies half in each medium. Let D_{1n} be the average normal component of displacement vector $\vec{D}$ to the bottom of the box in medium 1 and D_{2n} the average normal component of displacement vector $\vec{D}$ to the face of the box in medium 2. D_{1n} is the *inward normal.* By making the height of the cylinder h approaching zero, the contribution of the curved surface to the flux is taken as zero. Thus by Gauss's law, the total flux,

$$D_{2n}\,\Delta S - D_{1n}\,\Delta S = Q$$

where Q is total charge enclosed by the surface.

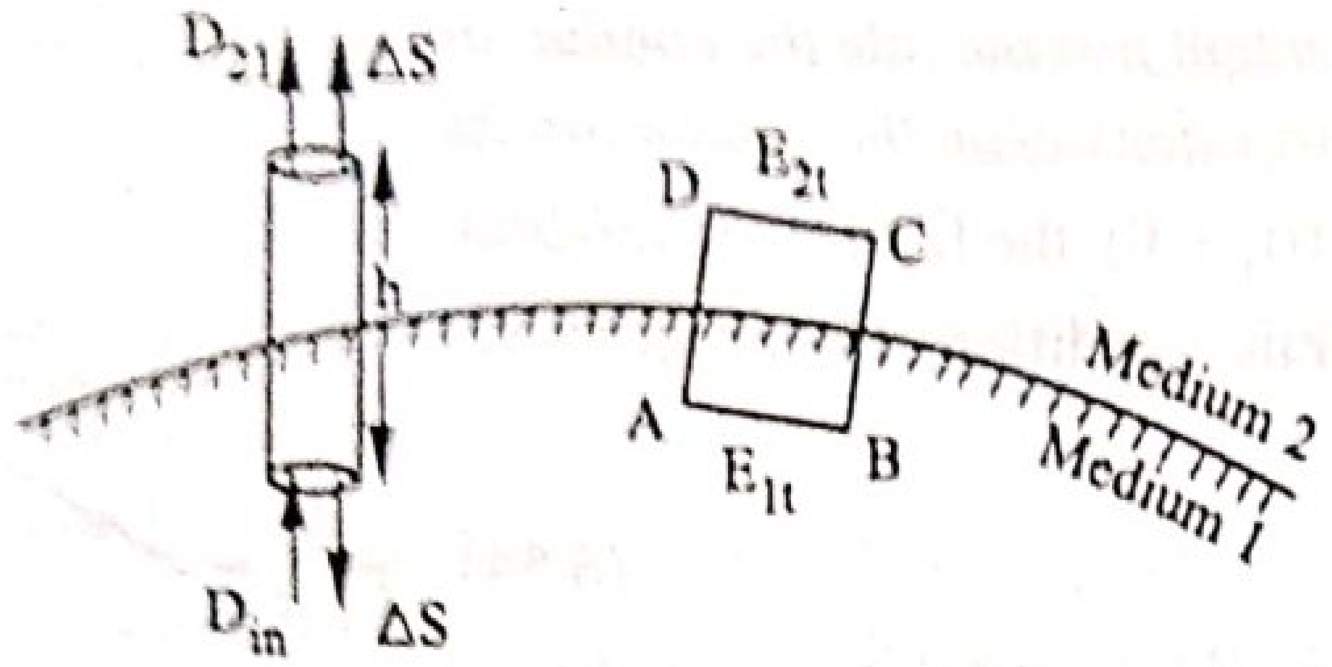

Fig. 6.16

The second term of left hand side of this equation is negative because D_{1n} and ΔS are oppositely directed. We can write,

$$D_{2n} - D_{1n} = \frac{Q}{\Delta S} = \sigma \qquad \text{...(6.50)}$$

where σ is the charge per unit area on the boundary of the two substances. *According to equation (6.50) the normal component of the displacement vector $\vec{D}$ changes at a charged boundary between two dielectrics by an amount equal to the surface charge density.*

If the boundary is free from charge $\sigma = 0$, then equation (6.50) reduces,

$$D_{2n} = D_{1n} \qquad \text{...(6.51)}$$

Thus, the normal component of displacement vector is continuous across the charge free boundary between two dielectrics.

We shall now use the idea that the electrostatics field is conservative, the integral of $\vec{E}$ around a closed path,

$$\int_{ABCD} \vec{E}.d\vec{l} = \int_{AB} \vec{E}.d\vec{l} + \int_{BC} \vec{E}.d\vec{l} + \int_{CD} \vec{E}.d\vec{l} + \int_{DA} \vec{E}.d\vec{l} = 0$$

By making the path length BC very small approaching zero, the work along the segments BC and DA of the path normal to the boundary is zero even though a finite electric field may exist normal to the boundary. Therefore, the line integral of $\vec{E}$ around ABCD rectangle is,

$$E_{1t}\,\Delta x - E_{2t}\,\Delta x = 0, \quad AB = CD = \Delta x$$

or $$E_{1t} = E_{2t} \qquad \text{...(6.52)}$$

Thus the tangential components of the electric field are the same on both sides of a boundary between two dielectrics. In other words, the *tangential electric field is continuous across such a boundary.*

2. Boundary conditions at the surface of a charged conductor

If one of the substances (medium or dielectric) considered above is a conductor, for example, if medium 1 is a conductor, then the electric field inside the conductor is zero, consequently, since there are no permanent dipoles in the conductors, the polarisation $\vec{P}$ must be zero, then according to relation $(\vec{D} = \varepsilon \vec{E} + \vec{P})$ $\vec{D}$ is also zero,

$$D_{1n} = 0$$

Thus equation (6.50) reduces,

$$D_{2n} = \sigma \qquad \text{...(6.53)}$$

This expresses the important result that the normal component of the displacement just outside the conductor is equal to the surface charge density on the conductor. As medium 1 is a conductor ($\sigma_1 \neq 0$) the field E_{1t} in medium 1 must be zero under static condition. Equation (6.52) reduces,

$$E_{2t} = 0 \qquad \ldots(6.54)$$

That is, the tangential component of the electric field just outside conductor (a dielectric conductor boundary) must be zero.

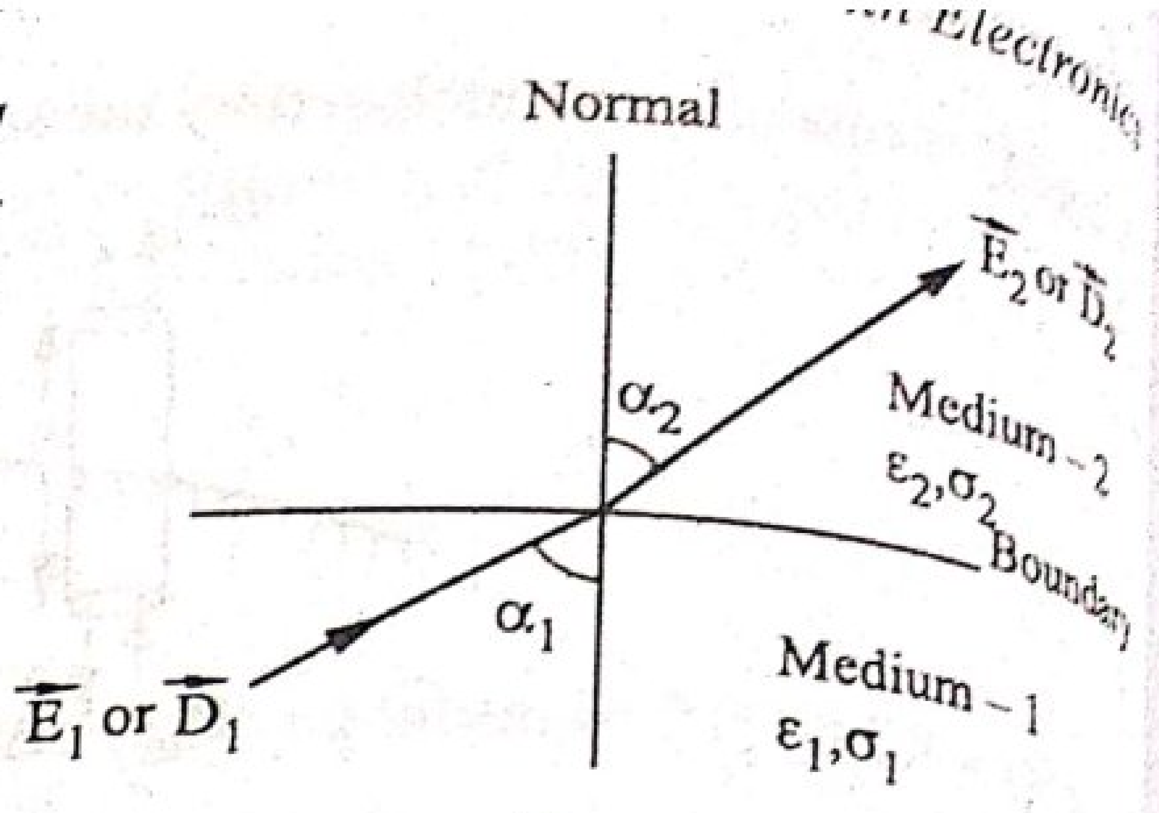

Fig.6.17

Scanned with CamScanner

EC-58

Subject : PHYSICS
Topic : Boundary Conditions / Boundary Relations.
Class : TDC-II & Paper – III
Date : 30-07-20
By : Dr. Vinay Kr. Singh (Physics)
NMV, Gorakothi (Siwan)

Scanned with CamScanner