Books for Study and Reference

1. Foundation of EMT – Third edition –John R. Reity, Frederick J. Milford and Robert W. Christy.
2. Electromagnetic theory – Prabir K. Basu and HrishikeshDhasmana.
3. Introduction to Electrodynamics– David J Griffiths.
4. Electromagnetic fields and waves– P.Lorrain and D.Corson.
5. Electrodynamics– B.P.Laud.

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Ans. (*a*) **Force on a dipole.** Consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2\,d\vec{l}$ apart, then dipole moment $\vec{p} = 2q\,d\vec{l}$

Uniform field. When such a dipole is placed in a uniform electric field $\vec{E}$ with its axis making an angle θ with the direction of $\vec{E}$ as shown, then

. Force on the charge $+q = +qE$

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i.e., in the direction of the field

Force on the charge $-q = -qE$

i.e., this force is equal and opposite to the force on the charge $+q$.

$\therefore$ Net force = 0

Hence the net translatory force on a dipole in a uniform electric field is zero.

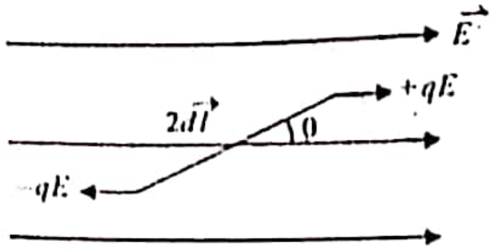

Fig. 4.19

Non-uniform field. When the field is non-uniform let it be $\vec{E}$ at the charge $-q$ lying at the position vector $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ and $\vec{E}'$ at the charge $+q$ lying at the position vector $\vec{r}' = \hat{i}(x + dx) + \hat{j}(y + dy) + \hat{k}(z + dz)$. Let E_x, E_y, E_z be the components of $\vec{E}$ and E_x', E_y', E_z' those of $\vec{E}'$. As the charge $+q$ is displaced from the charge $-q$ by a vector distance

$$\vec{r}' - \vec{r} = 2\vec{dl} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$$

the dipole moment

$$\vec{p} = 2q\vec{dl} = q(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

Further

$$E_x' = E_x + \left(\frac{\partial E_x}{\partial x}\right)dx + \left(\frac{\partial E_x}{\partial y}\right)dy + \left(\frac{\partial E_x}{\partial z}\right)dz$$

$\therefore$ Net force on the dipole in the X-direction

$$F_x = qE_x' - qE_x = q\left[\left(\frac{\partial E_x}{\partial x}\right)dx + \left(\frac{\partial E_x}{\partial y}\right)dy + \left(\frac{\partial E_x}{\partial z}\right)dz\right]$$

$$= q\,(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)\,.\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_x}{\partial y} + \frac{\partial E_x}{\partial z}\right)$$

$$= q\,2\vec{dl}\,.\,\vec{\nabla}\,E_x = \vec{p}\,.\,\vec{\nabla}\,E_x$$

Similarly net force on the dipole along the Y and Z direction is given by

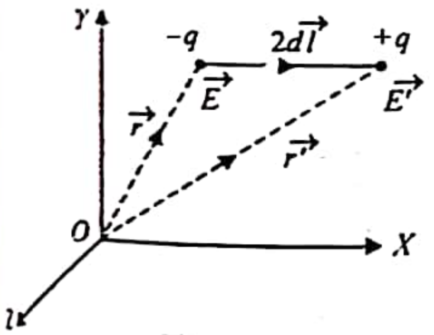

Fig. 4.20

$$F_y = \vec{p}\,.\,\vec{\nabla}\,E_y$$

and

$$F_z = \vec{p}\,.\,\vec{\nabla}\,E_z$$

$\therefore$ Total force on the dipole

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$= (\vec{p}\,.\,\vec{\nabla})\,(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})$$

$$= (\vec{p}\,.\,\vec{\nabla})\,\vec{E}$$

$(\vec{p}\,.\,\vec{\nabla})$ is a scalar differential operator given by

$$\left(p_x\frac{\partial}{\partial x} + p_y\frac{\partial}{\partial y} + p_z\frac{\partial}{\partial z}\right)$$

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Torque. It has been shown that the net translatory force on a dipole in a uniform electric field is zero. The two forces do not act a long the same straight line and constitute a couple, the moment of which is given by

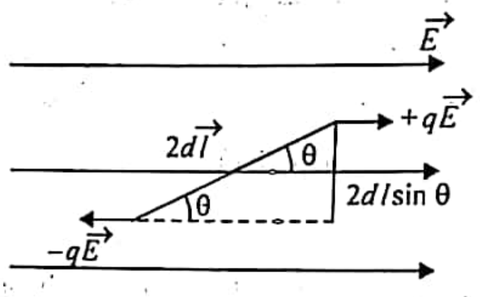

Fig. 4.21

$$qE\,2dl \sin\theta = pE\sin\theta$$

Since $\vec{E}$ and $\vec{P}$ are vector quantities.

$$pE\sin\theta = \vec{p}\times\vec{E}$$

∴ Torque on the dipole

$$\tau = \vec{p}\times\vec{E}$$

The torque = 0 when $\vec{P}$ is parallel to $\vec{E}$.

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EC-57

Subject : PHYSICS

Topic : Torque on a dipole in non-uniform electric field.

Class : TDC-II & paper - IV

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