Books for Study and Reference

1. Foundation of EMT – Third edition –John R. Reity, Frederick J. Milford and Robert W. Christy.
2. Electromagnetic theory – Prabir K. Basu and HrishikeshDhasmana.
3. Introduction to Electrodynamics– David J Griffiths.
4. Electromagnetic fields and waves– P.Lorrain and D.Corson.
5. Electrodynamics– B.P.Laud.

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2.12. ELECTRIC DIPOLE

A system of two equal and opposite point charges separated by a small distance is called an **electric dipole.**

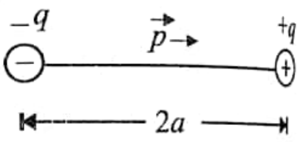

Fig. 2.22

Fig. 2.22 shows an electric dipole consisting of two equal and opposite charges ($-q$, $+q$) separated by a small distance '$2a$'. The line joining the charges is called *dipole axis.* The length of the dipole $(\overrightarrow{2a})$ is a *vector* whose direction is from the charge $-q$ to charge $+q$. The total charge of the dipole $= -q + q = 0$. But the electric field due to electric dipole is not zero. It is because the charges $-q$ and $+q$ are separated by some distance and the resultant field due to these charges is not zero.

Many molecules (*e.g.* HCl, H_2O etc.) behave as electric dipoles. In these molecules, the centre of positive charge does not coincide with the centre of negative charge. The result is that one end of the molecule is positively charged and the other end is equally negatively charged, although the molecule is neutral. Consequently, the molecule behaves as an electric dipole.

2.13. DIPOLE MOMENT ($\vec{p}$)

The dipole moment is a measure of the strength of electric dipole.

The dipole moment of an electric dipole is a vector whose magnitude is equal to the product of either charge and length of the dipole i.e.

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$$\vec{p} = q(2\vec{a})$$

The direction of $\vec{p}$ is along dipole axis from $-q$ to $+q$. The SI unit of dipole moment *is coulomb-metre* (Cm). The dimensional formula of dipole moment is $[M^0 LTA]$.

Ideal or point dipole. The magnitude of dipole moment of electric dipole is $p = q \times 2a$. If size $2a \to 0$ and charge $q \to \infty$ such that dipole moment remains the same, the resulting dipole is called ideal or point dipole.

A dipole of negligibly small size is called an ideal or point dipole.

Although the point dipole is an idealisation, it is useful approximation to a real dipole whose size is small compared with distances to other charge distributions.

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17.20 ELECTRIC QUADRUPOLE

A linear electric quadrupole consists of two electric dipoles placed end to end along the same line.

As shown in Fig. 17.18, *OA* is one dipole having a charge $-q$ at *O* and $+q$ at *A* and *OB* is the second dipole having a charge $-q$ at *O* and $+q$ at *B*. *Thus the total charge at O is $-2q$ and the total electric charge or the monopole moment of the charge distribution of the system is zero.*

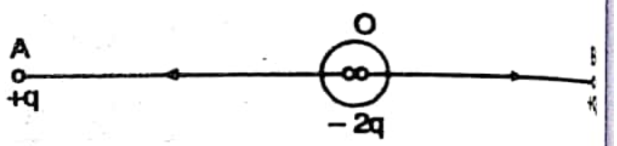

Fig. 17.18

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17.21 ELECTRIC FIELD $\vec{E}$ DUE TO QUADRUPOLE

The electric potential due to quadrupole at any point situated far off is given by

$$V_{(r)} = \frac{qd^2}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 1) \qquad (\because \text{ Eq } (ii) \text{ of article } 17.20)$$

By making use of relation, $\vec{E} = -\vec{\nabla}V$, we can calculate the electric field.

The X-component in cartesian co-ordinate system is

$$|\vec{E}_x| = -\frac{\partial V}{\partial x} = -\frac{qd^2}{4\pi\epsilon_0}\cdot\frac{\partial}{\partial x}\left(\frac{3\cos^2\theta - 1}{r^3}\right)$$

$$= -\frac{qd^2}{4\pi\epsilon_0}\cdot\frac{\partial}{\partial x}\left(\frac{3x^2}{r^5} - \frac{1}{r^3}\right) \qquad (\text{Since } x = r\cos\theta)$$

$$= -\frac{qd^2}{4\pi\epsilon_0}\left[\frac{6x}{r^5} + \frac{\partial}{\partial x}(r^{-5}).3x^2 - \frac{\partial}{\partial x}(r^{-3})\right]$$

$$= -\frac{qd^2}{4\pi\epsilon_0}\left[\frac{6x}{r^5} + \frac{\partial}{\partial x}(r^{-5})\frac{\partial}{\partial x}3x^2 - \frac{\partial}{\partial_r}(r^{-3})\frac{\partial_r}{\partial x}\right]$$

But $\qquad r^2 = x^2 + y^2 + z^2$

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$$2r\frac{\partial_r}{\partial x} = 2x \text{ OR } \frac{\partial_r}{\partial x} = \frac{x}{r}$$

$$\left|\vec{E}_x\right| = -\frac{qd^2}{4\pi \epsilon_0}\left[\frac{6x}{r^5} + 3x^2(-5r^{-6})\frac{x}{r} + 3r^{-4}\frac{x}{r}\right]$$

$$= -\frac{qd^2}{4\pi \epsilon_0}\left[\frac{6x}{r^5} - \frac{15x^3}{r^7} + \frac{3x}{r^5}\right]$$

$$= \frac{3qd^2}{4\pi \epsilon_0 r^5}\left(\frac{5x^3}{r^2} - 3x\right)$$

$$= \frac{3qd^2}{4\pi \epsilon_0 r^5}\left(\frac{5r^3\cos^3\theta}{r^2} - 3r\cos\theta\right) \qquad \text{(since } x = r\cos\theta\text{)}$$

$$= \frac{3qd^2}{4\pi \epsilon_0 r^5}(5r\cos^3\theta - 3r\cos\theta)$$

$$= \frac{3qd^2}{4\pi \epsilon_0 r^4}(5\cos^3\theta - 3\cos\theta)$$

Similarly Y-Component of the field is obtained as

$$\left|\vec{E}_Y\right| = -\frac{\partial V}{\partial y} = -\frac{qd^2}{4\pi \epsilon_0}\frac{\partial}{\partial y}\left(\frac{3\cos^2\theta - 1}{r^3}\right)$$

...ing in the same way, we get,

$$\left|\vec{E}_Y\right| = \frac{3qd^2}{4\pi \epsilon_0 r^5}(5\cos^2\theta - 1)\, y$$

...Z-component of the field as,

$$\left|\vec{E}_Z\right| = \frac{3qd^2}{4\pi \epsilon_0 r^5}(5\cos^2\theta - 1)\, z$$

Therefore, the magnitude of the intensity,

$$\left|\vec{E}\right| = \sqrt{\left|\vec{E}_x\right|^2 + \left|\vec{E}_y\right|^2 + \left|\vec{E}_z\right|^2}$$

$$= \frac{3qd^2}{4\pi \epsilon_0}\left[\frac{1}{r^8}(5\cos^3\theta - 3\cos\theta)^2 + \frac{1}{r^{10}}(5\cos^2\theta - 1)^2 y^2 + \frac{1}{r^{10}}(5\cos^2\theta - 1)^2 z^2\right]^{1/2}$$

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$$= \frac{3qd^2}{4\pi \in_0 r^4}\left[(5\cos^3\theta - 3\cos\theta)^2 + \frac{1}{r^2}(5\cos^2\theta - 1)^2(y^2 + z^2)\right]^{1/2}$$

$$= \frac{3qd^2}{4\pi \in_0 r^4}\left[(5\cos^3\theta - 3\cos\theta)^2 + (5\cos^2\theta - 1)(1 - \cos^2\theta\right]^{1/2}$$

Since $y^2 + z^2 = r^2 - x^2 = r^2 - r^2\cos^2\theta$ ($\because x = r\cos\theta$)

$\therefore$ $y^2 + z^2 = r^2(1 - \cos^2\theta)$

Solving squares and product, we get

$$|\vec{E}| = \frac{3qd^2}{4\pi \in_0 r^4}\left[5\cos^4\theta - 2\cos^2\theta + 1\right]^{1/2}$$

Thus, the electric field of a linear quadrupole varies inversely as the fourth power of the distance.

The following important cases arise:

Case 1 : Field on the axis of quadrupole.

i.e. if $\theta = 0$, the field is maximum

$$|\vec{E}|_{max} = \frac{6qd^2}{4\pi \in_0 r^4} = \frac{3Q_d}{4\pi \in_0 r^4}$$

(as $2qd^2 = Q_d$ = the quardupole moment)

Case 2 : Field along the equatorial line.

i.e. if $\theta = 90°$, the field is minimum

$$|\vec{E}|_{min} = \frac{3qd^2}{4\pi \in_0 r^4} = \frac{3Q_d}{8\pi \in_0 r^4}$$

Thus, the field on the axis of the quadrupole is double to that of the field at the same distance on equatorial line.

Case 3 : Field on the – ve X-axis:

i.e. $\theta = 180°$, then, the field is again maximum, but in a direction of negative *X*-axis.

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EC-55

~~Ele~~ Subject : PHYSICS

Topic : Electric field due to Quadrupole.

Class : BSc.-II, Paper-IV

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